Viscous Aerodynamic Analysis of an Oscillating Flat-Plate Airfoil

Linda M. Schroeder* and Sanford Fleeter†
Purdue University, West Lafayette, Indiana

Abstract

CONSIDERABLE progress has been made in the prediction of the unsteady aerodynamics of oscillating airfoils. These analyses are typically limited to inviscid potential flows, with the unsteady flow assumed to be a small perturbation to the mean flow and the Kutta condition imposed. By considering the airfoils to be zero-thickness flat plates at zero mean incidence, the steady and unsteady flowfields are uncoupled, with the steady flow being uniform and parallel.

In this paper, an analysis is developed that models the unsteady aerodynamics of an harmonically oscillating flat-plate airfoil, including the effects of mean flow incidence angle, in an incompressible laminar flow at moderate values of the Reynolds number. The unsteady viscous flow is assumed to be a small perturbation to the steady viscous flowfield. The nonuniform and nonlinear steady flowfield is described by the Navier-Stokes equations and is independent of the unsteady flow. The small-perturbation unsteady viscous flow is described by a system of linear partial differential equations that are coupled to the steady flowfield, thereby modeling the strong dependence of the unsteady aerodynamics on the steady flow. Solutions for both the steady and unsteady viscous flowfields are obtained by a locally analytical method in which the discrete algebraic equations representing the flowfield equations are obtained from analytical solutions in individual local grid elements.

The locally analytical method for steady two-dimensional fluid flow and heat-transfer problems was initially developed by Chen et al.[1,2] They have shown that it has several advantages over finite-difference and finite-element methods, including being less dependent on grid size, with the system of algebraic equations relatively stable. Also, since the solution is analytical, it is differentiable and is a continuous function.

Contents

For harmonic time dependence at a frequency ω, the nondimensional continuity and Navier-Stokes equations in terms of the vorticity $\bar{\zeta}$ and the stream function $\bar{\psi}$ are

$$\nabla^2\bar{\zeta} = \bar{\zeta}_{xx} + \bar{\zeta}_{yy} = Re(k\bar{\zeta}_t + \bar{u}\bar{\zeta}_x + \bar{v}\bar{\zeta}_y) \tag{1a}$$

$$\nabla^2\bar{\psi} = -\bar{\zeta} \tag{1b}$$

where $\bar{\zeta} = \bar{v}_x - \bar{u}_y$, $\bar{u} = \bar{\psi}_y$, $\bar{v} = -\bar{\psi}_x$, $Re = U_\infty C/\nu$ denotes the Reynolds number based on the airfoil chord, and $k = \omega C/U_\infty$ is the reduced frequency.

Presented as Paper 88-0130 at the AIAA 26th Aerospace Sciences Meeting, Reno, NV, Jan. 11-14, 1988; received Feb. 29, 1988; synoptic received Aug. 15, 1988. Full paper available at AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, $4.00; hard copy, $9.00. **Remittance must accompany order.**

*AFRAPT Trainee, Thermal Sciences and Propulsion Center, School of Mechanical Engineering (currently Associated Aerodynamics Engineer, Sundstrand TurboMach). Member AIAA.

†Professor and Director, Thermal Sciences and Propulsion Center, School of Mechanical Engineering. Associate Fellow AIAA.

The equations describing the steady and unsteady viscous flows are determined by decomposing the flowfield into steady and small-perturbation harmonic unsteady components. For the unsteady flow, the second-order terms are neglected as small compared to the first-order terms.

The coupled nonlinear partial differential equations describing the steady flowfield are independent of the unsteady flow as shown in Eq. (2). The vorticity equation is nonlinear, with the stream function described by a linear Poisson equation that is coupled to the vorticity equation through the vorticity source term. The pressure also is described by a linear Poisson equation, with the source term dependent on the steady flowfield,

$$\nabla^2\zeta = Re(U\zeta_x + V\zeta_y) \tag{2a}$$

$$\nabla^2\psi = -\zeta \tag{2b}$$

$$\nabla^2 P = -2(U_x V_y - V_x U_y) \tag{2c}$$

where U and V are the steady chordwise and normal velocity components.

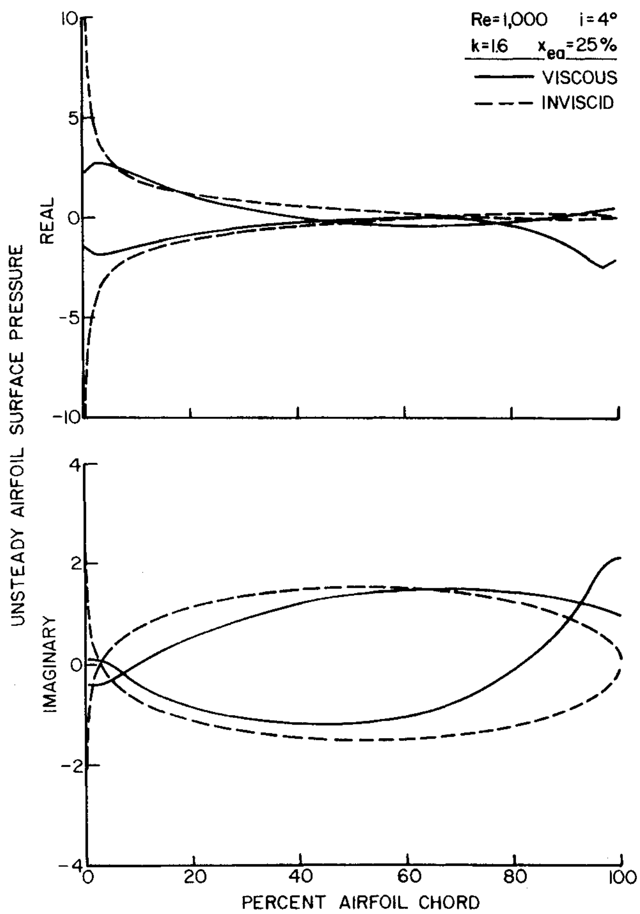

Fig. 1 Unsteady airfoil surface pressures for *Re* = 1000 and 4 deg incidence.

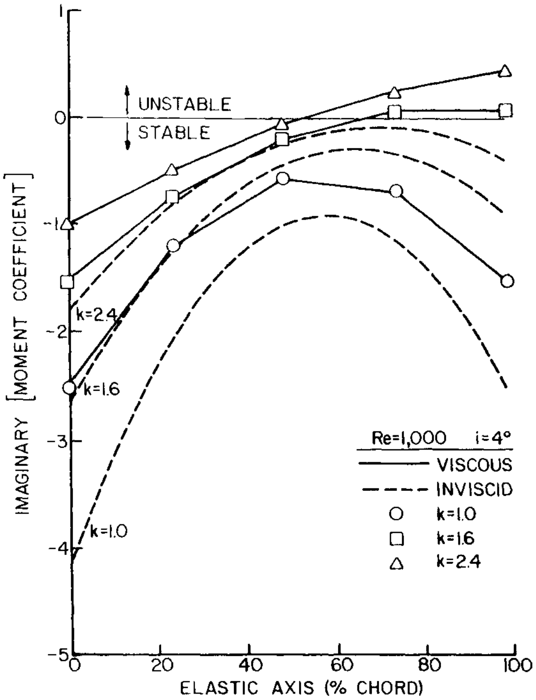

Fig. 2 Variation of the imaginary aerodynamic moment coefficient with elastic axis for $Re = 1000$ and 4 deg incidence.

The coupled linear partial differential equations describing the unsteady harmonic flowfield are given in Eq. (3). The unsteady flow is coupled to the steady flowfield. In particular, in both the unsteady vorticity transport and pressure equations, the coefficients are dependent on the steady flowfield with the unsteady stream function coupled to the solution for the unsteady vorticity.

$$\nabla^2 \xi = Re(ki\xi + U\xi_x + V\xi_y + u\zeta_x + v\zeta_y) \tag{3a}$$

$$\nabla^2 \psi = -\xi \tag{3b}$$

$$\nabla^2 p = -2[(u_x V_y + v_y U_x) - (v_x U_y + u_y V_x)] \tag{3c}$$

where $i = \sqrt{-1}$, and u and v denote the unsteady perturbation chordwise and normal velocity components.

The steady flow boundary conditions specify no slip between the fluid and the surface and that the velocity normal to the surface is zero. For the unsteady flow, the velocity of the fluid must be equal to that of the surfaces and the unsteady chordwise velocity component must satisfy a no-slip boundary condition. For a flat-plate airfoil, executing small-amplitude harmonic torsion mode oscillations about an elastic axis location at x_{ea} measured from the leading edge, the linearized normal velocity boundary condition in Eq. (4) is applied on the mean position of the oscillating airfoil,

$$v(x, 0) = \alpha[ik(x - x_{ea}) + U_o]e^{it} \tag{4}$$

where α is the amplitude of oscillation.

Locally analytical solutions for the unsteady and steady viscous flowfields then are developed. In this method, the discrete algebraic equations that represent the aerodynamic equations are obtained from analytical solutions in individual local grid elements. This is accomplished by dividing the flowfield into computational grid elements. In each element, the nonlinear convective terms of the steady Navier-Stokes equations are locally linearized. The nonlinear character of the steady flowfield is preserved as the flow is only locally linearized, that is, independently linearized in individual grid elements. Analytical solutions to the linear equations describing both the steady and unsteady flowfields in each element then are determined. The solution for the complete flowfield is obtained through the application of the global boundary conditions and the assembly of the locally analytic solutions.

This unsteady viscous flow model and locally analytical solution are used to investigate the effects of Reynolds number, mean flow incidence angle, and reduced frequency on the unsteady aerodynamics of an harmonically oscillating airfoil. Predictions are obtained on a 50×35 rectangular grid with $\Delta x = 0.025$ and $\Delta y = 0.025$ and 21 points located on the airfoil. The convergence criteria for the stream function iterations are both 10^{-4}, with the vorticity tolerance being 5×10^{-2}. The tolerances for the pressure iterations are 10^{-6} and 10^{-5} for the internal and external iterations, respectively. The computational time averaged 440 CPU on the Cyber 205, with an average of 160 iterations for the stream function and vorticity solutions and an additional 160 iterations for the pressure solution.

The chordwise distributions of the complex unsteady pressure on the individual surfaces of an oscillating airfoil at four degrees of incidence and a Reynolds number of 1000 is presented in Fig. 1. The corresponding classical inviscid Theodorsen prediction[3] is also shown. Viscosity has a large effect on the complex unsteady surface pressures, particularly the real part, over the front part of the airfoil. In particular, one difference between the two solutions is that the viscous solution is finite at the leading edge, whereas the inviscid solution is singular.

The torsion mode flutter stability of an airfoil is determined by the imaginary part of the unsteady aerodynamic moment in Eq. (5). Thus, Fig. 2 shows the airfoil stability as a function of the elastic axis location, with the reduced frequency as parameter at a Reynolds number of 1000 for an incidence angle of 4 deg, together with Theodorsen's inviscid zero incidence results. Viscous effects are seen to generally decrease the relative stability of the airfoil at all elastic axis locations, with the largest relative decrease in airfoil stability associated with the lower reduced frequency:

$$C_M = \frac{M}{\frac{1}{2}\rho C^2 U^2 k^2 \pi} = \frac{\int_0^1 (p_{\text{lower}} - p_{\text{upper}})(x - x_{ea})\, dx}{\frac{1}{4}\rho C^2 U^2 k^2 \pi} \tag{5}$$

Acknowledgment

This research was sponsored, in part, by the Air Force Office of Scientific Research.

References

[1]Chen, C. J., Naseri-Neshat, H., and Ho, K. S., "Finite Analytic Numerical Solution of Heat Transfer in Two-Dimensional Cavity Flow," *Journal of Numerical Heat Transfer,* Vol. 4, 1981, pp. 179–197.

[2]Chen, C. J. and Yoon, Y. H., "Finite Analytic Numerical Solution of Axisymmetric Navier-Stokes and Energy Equations," *Journal of Heat Transfer,* Vol. 5, Aug. 1983, pp. 639–645.

[3]Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," NACA TR-496, 1935.